EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

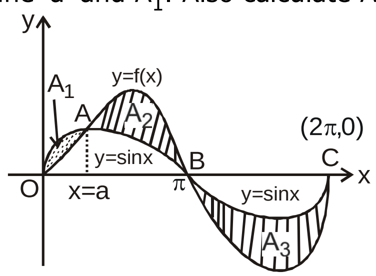
1. A polynomial function f(x) satisfies the condition $f(x + 1) = f(x) + 2x + 1$. Find f(x) if $f(0) = 1$. Find also the equations of the pair of tangents from the origin on the curve $y = f(x)$ and compute the area enclosed by the curve and the pair of tangents.

2. Find the equation of the line passing through the origin and dividing the curvilinear triangle with vertex at the origin, bounded by the curves $y = 2x - x^2$, $y = 0$ and $x = 1$ into two parts of equal area.

3. Consider the curve $y = x^n$ where $n > 1$ in the 1st quadrant. If the area bounded by the curve, the x-axis and the tangent line to the graph of $y = x^n$ at the point (1, 1) is maximum then find the value of n.

4. Consider the collection of all curve of the form $y = a - bx^2$ that pass through the point (2, 1), where 'a' and 'b' are positive constants. Determine the value of 'a' and 'b' that will minimize the area of the region bounded by $y = a - bx^2$ and x-axis. Also find the minimum area.

5. In the adjacent figure, graphs of two functions y = f(x) and y = sinx are given. y = sinx intersects, y = f(x) at A(a, f(a)); B(π, 0) and C(2π, 0). A_i (i = 1, 2, 3) is the area bounded by the curves y = f(x) and y = sinx between x = 0 and x = a; i = 1, between x = a and x = π; i = 2, between x = π and x = 2π; i = 3. If $A_1 = 1 - \sin a + (a-1)\cos a$, determine the function f(x). Hence determine 'a' and A_1. Also calculate A_2 and A_3.



6. Show that the area bounded by the curve $y = \frac{\ln x - c}{x}$, the x-axis and the vertical line through the maximum point of the curve is independent of the constant c.

7. For what value of 'a' is the area of the figure bounded by the lines, $y = \frac{1}{x}$, $y = \frac{1}{2x-1}$, x = 2 and x = a equal to $\ell n \frac{4}{\sqrt{5}}$?

8. Compute the area of the loop of the curve $y^2 = x^2 [(1 + x)/(1 - x)]$.

9. For the curve $f(x) = \frac{1}{1+x^2}$, let two points on it are $A(\alpha, f(\alpha))$, $B\left(-\frac{1}{\alpha}, f\left(-\frac{1}{\alpha}\right)\right)$ $(\alpha > 0)$. Find the minimum area bounded by the line segments OA, OB and f(x), where 'O' is the origin.

10. Let 'c' be the constant number such that c > 1. If the least area of the figure given by the line passing through the point (1, c) with gradient 'm' and the parabola $y = x^2$ is 36 sq. units find the value of $(c^2 + m^2)$.

11. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines x = 0, y = 0 and x = π/4. Prove that for n > 2, $A_n + A_{n-2} = 1/(n - 1)$ and deduce that $1/(2n + 2) < A_n < 1/(2n - 2)$.

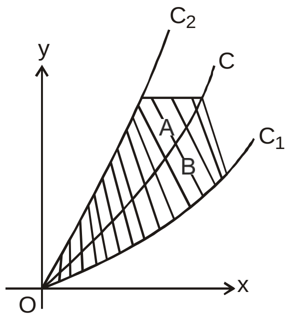
12. If f(x) is monotonic in (a, b) then prove that the area bounded by the ordinates at x = a; x = b; y = f(x) and y = f(c), c ∈ (a, b) is minimum when $c = \frac{a+b}{2}$. Hence if the area bounded by the graph of $f(x) = \frac{x^3}{3} - x^2 + a$, the straight lines x = 0, x = 2 and the x-axis is minimum then find the value of 'a'.

13. Consider the two curves $C_1 : y = 1 + \cos x$ and $C_2 : y = 1 + \cos(x - \alpha)$ for $\alpha \in (0, \pi/2)$; $x \in [0, \pi]$. Find the value of α, for which the area of the figure bounded by the curves C_1, C_2 and x = 0 is same as that of the figure bounded by C_2, y = 1 and x = π. For this value of α, find the ratio in which the line y = 1 divides the area of the figure by the curves C_1, C_2 and x = π.

14. Find the whole area included between the curve $x^2y^2 = a^2(y^2 - x^2)$ and its asymptotes (asymptotes are the lines which meet the curve at infinity).

15. For what values of a ∈ [0, 1] does the area of the figure bounded by the graph of the function y = f(x) and the straight lines x = 0, x = 1 and y = f(a) is at a minimum and for what values it is at a maximum if $f(x) = \sqrt{1-x^2}$. Find also the maximum and the minimum areas.

16. Let C_1 and C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" the region between C_1 and C_2, if for each point P of C, the two shaded regions A and B shown in the figure have equal areas. Determine the upper curve C_2, given that the bisecting curve C has the equation $y = x^2$ and that the lower curve C_1 has the equation $y = x^2/2$.



17. Given $f(x)=\int_0^x e^t(\ln\sec t - \sec^2 t)\,dt$; $g(x)=-2e^x \tan x$. Find the area bounded by the curves $y = f(x)$ and $y = g(x)$ between the ordinates $x = 0$ and $x = \frac{\pi}{3}$.